Reconstruction of Wind Turbine Blade Geometry and Shape Matching of Airfoil Profiles to Point Clouds

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Abstract. Wind Turbine Blades have a limited operation life due to safety and performance concerns. At the end of this cycle, these massive composite structures still retain more than sufficient structural capacity for architectural and infrastructure re-use applications. Re-Wind research group develops ideas and technologies for at scale solutions of accumulating blade waste problem. Blade Machine (BM) is the software workflow used in construction of reliable digital twins for these objects. We share the results of turbine blade geometry reconstructions from 3D point clouds using a knowledge-based approach. With necessary adjustments the described method can be generalized for application in a vast array of objects in the built environment for surveying or re-use.

1. Introduction

Wind-turbine blades are monolithic fiber-reinforced polymer (FRP) structures built to withstand immense loads applied by wind. As these components perform in high-speed motion, they are operated for a restricted lifespan to ensure predictable performance. At the end of this period, the wind blades are decommissioned but retain significant residual structural capacity for lower demand applications.

Recycling of Turbine Blades are associated with such high energy and labor costs that the current practice is abandonment in landfills, incineration followed by landfill, or in the best-case cement kiln co-processing. This latter happens in limited locations and at a small scale. All these methods are energy inefficient and contribute to CO_2 emissions.(Deeney et al., 2021) Considering the expected volume of cumulative blade waste is projected to reach 43 million tons,(Liu and Barlow, 2017) adaptive reuse offers an efficient alternative by prolonging the life-cycle of turbine blades. (Nagle et al., 2022)

The Re-Wind project focuses on developing adaptive re-use ideas for the de-commissioned wind blades. Adaptive re-use ideas such as high voltage electrical power structures (Alshannaq et al., 2019) pedestrian bridges (Leahy et al., 2021) and housing (Bank et al., 2018) (Gentry et al., 2020) have been proposed by the Re-Wind team. Unlike incineration and disposal, re-use applications require accurate and intricate CAD, structural engineering, and CAM models to facilitate the re-design and re-engineering processes.

The collection of tools and methods used to build these models is referred to as Blade Machine (BM). This paper focuses on the problem of building surface-based virtual models from point clouds of turbine blades.

Turbine blade manufacturing sector is highly competitive. Most designers and manufacturers keep specifications for their products secret. By the time the blades are decommissioned, specifications and shop drawings are often lost as they are not retained in public repositories. As Figure 1 indicates, one data model Re-Wind project has developed relies on structured manufacturing documentation. Yet due to the scarcity of this type of information, this method is not globally applicable.

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The Blade Machine workflow starts downstream of the acquisition of point clouds. Due to their scale and non-uniform geometry, objects like wind blades fall into an underdeveloped niche regarding reality capture and Scan-to-BIM research.

For practical purposes and modularity BM is articulated in four phases. (Figure 1.) These are:

- Phase 1: Point cloud orientation, cleanup, airfoil recognition, AeroDyn output and skeleton model.
- Phase 2: Structural topology assertation, surface model generation and structural segmentation.
- Phase 3: Calculation of composite section properties for engineering analysis and back-calculation of section stresses from external 3D frame analysis.
- Phase 4: Mesh generation for finite element analysis. Generation of solid models for re-fabrication tasks such as CNC blade cutting and drilling.

This paper focuses on the considerations and methods of the Phase 1 of Blade Machine implementation – focusing on digital reconstruction of complex geometry.

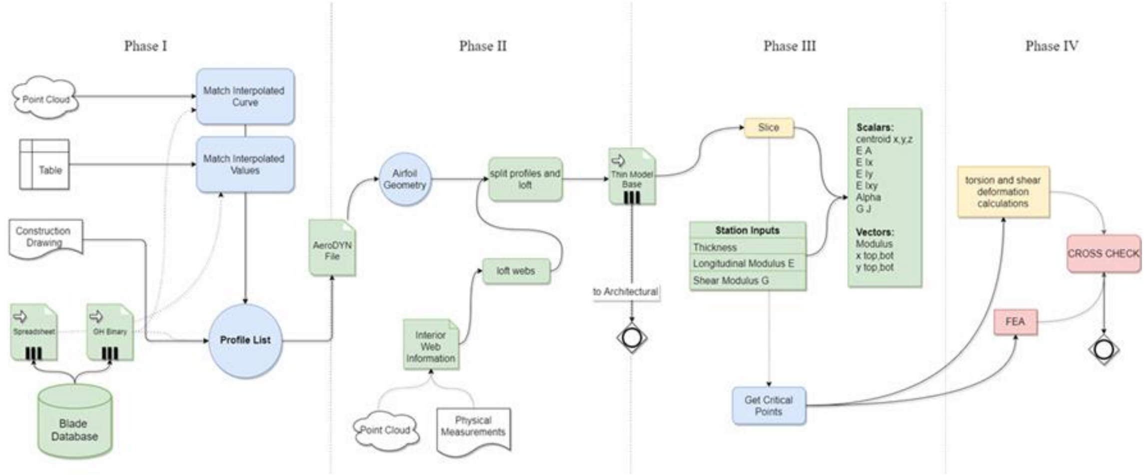

Figure 1: Blade Machine workflow.

2. Concepts and Approach

An initial method for digital reconstruction of a wind-blade has been presented in Tasistro-Hart et al. (2019). This method relied on genetic optimization algorithms to find best-fit matches of airfoil sections in synthetic point clouds generated from digital models. The method worked well in the tests cases but proved unreliable noisy real-world point clouds obtained with laser scanning and photogrammetry.

Some of these artifacts in the sample scans were noisy data, non-uniform sampling due to skewed perspective and loss of resolution, missing data due to occlusion, and adjacent but otherwise unrelated objects captured. A second type of artifact that can be introduced in post-processing is erroneous alignment of partial point clouds.

Another change in BM software, after the initial publication, was the definition of the geometry around an internal curvilinear reference axis defined by section geometry. This facilitates the formatting of the information to industry standards such as AeroDyn. (Moriarty and Hansen, 2005, Jonkman et al., 2015)

2.1 Point Clouds

Given the limited information available regarding the fabrication geometry of turbine blades, reality capture methods are used to obtain product geometry. Common methods for objects of similar scale are stereo-photogrammetry, Structure from Motion (SFM), and laser or lidar scanning. These survey models provide point clouds and can contain information ranging from object color to surface normals.

Point clouds are useful tools to store information collected in the field about the “as is” conditions of large objects. No-Contact methods for their acquisition are cheap and fast to use and can produce highly detailed results. Point clouds are data intense; this makes them require large storage, information transfer, and processing capabilities. There are many methods used to reconstruct the geometries found within point clouds. (Berger et al., 2017)

Our choice of CAD software was Rhinoceros 3D, because of its easily scriptable add-on Grasshopper. Grasshopper 3D provides an intuitive Visual Programming Language interface with the CAD software. Furthermore, common open-source point cloud processing software such as CloudCompare and MeshLab can interface with this environment directly through components.

Workarounds, plugins, and additional scripting have been kept to a minimum to maintain portability. While their use for algorithms requiring iterations and looping were inevitable in the general BM workflow, the operations described in this paper can be implemented without resorting to such extensions.

Segmentation is an umbrella term for methods used to distinguish homogeneous clusters of points that are geometrically significant. Again, there are many emerging methods regarding segmentation. (Nguyen and Le, 2013)

To achieve a dimensional reduction of the segmentation problem, our method leverages product knowledge specific to airfoils within the context of turbine blade manufacturing. We adopt a hybrid (graph and geometry based) shape retrieval method as the internal feature around which our sections are searched for (blade axis) is not a physical entity that can be captured.(Tangelder and Veltkamp, 2007)

The difficulty in parametrization of these sections, coupled with the lack of information available for creation of training and testing sets, precludes the application of other established methods of segmentation.(Agapaki and Brilakis, 2022)

We use a proxy database of pre-encoded airfoils to match. The blade database lets the user choose to develop continuous geometries through spline interpolations. The possibility of using NURBS geometry allows for non-linear interpolation, smaller model sizes, and continuous surface models depending on the needs of the downstream use.

2.2 Turbine Blade Geometry

Blade turbine geometry presents many challenges; both in construction and the survey of the artifact. Overall, these are cantilever beam type geometries with variable sections. The length of the object is markedly larger than the other dimensions. Most of the large-scale re-use proposals take advantage of the length of these objects. This characteristic lends itself to a dimensional reduction where we consider the overall geometry as a series of sections lying along a curve developing in the major dimension.

We assume the scanned object to be monolithic. This is true for global reconstructions but does not account for internal or subsurface components. These components are populated into the model in later phases of the workflow to build high fidelity digital proxies. The exterior is a concatenation of cylindrical, ruled, or lofted surfaces. Sections found in the initial phase are utilized to recreate these considering construction methods and aerodynamic geometries. Variable section segmenting and rebuilding is a transferrable workflow to different applications. Some of these are already applied in MEP construction tracking. (Bosché et al., 2014)

For the sake of brevity internal structures of turbine blades will not be detailed in this article. Yet whenever possible these scans provide a valuable alternative to destructive testing and are integrated to the model in later stages of BM workflow.

2.3 Airfoils:

The cross-sectional shapes of our turbines are referred to as Airfoils. These shapes produce an aerodynamic force while interacting with fluids in relative motion. These curves confer lift and drag properties to the blade combined with the twist governing the angle of attack along its length. (Bertin and Smith, 1998, Biot, 1942)

Airfoils are characterized by a rounder Leading Edge (LE) and a sharper Trailing Edge (TE). The names derive from the order in which these sections are designed to encounter oncoming airflow. (Hau, 2013)

Two geometrically significant curves span between these two points. The straight line running across these is referred to as the airfoil's chord line and is the basis for measuring the relative chord length and the thickness. The second one is the camber line. The difference in the shell geometry between the two halves delimited by the camber line in non-symmetric airfoils creates a pressure difference between the two sides. These sides are named the Low Pressure (LP) side and High Pressure (HP) side. The camber line traverses the middle points between the HP and LP sides on stations developed orthogonally to the chord line. The chord line can intersect the outer shell, and the camber line is contained within the airfoil save for the intersections with the TE and LE points. And while the former is the directrix of an orthogonal basis the latter is the median curve used to distinguish between HP and LP sides. The two lines are coincident in symmetric sections. Low pressure side presents a convex form while the high-pressure side is convex on LE and concave on TE portions.

In the spatial development of a turbine blade, the chord lines —or their projections to the world X-Y plane— generally present a sequential rotation along the blade axis to give desired angles of attack (AoA) to the different sections along the length of the blade. This is called the Twist of the blade.

Airfoils are classified, published, or held as proprietary knowledge by diverse organizations. Depending on the compiling or publishing entity's specifications, airfoils are grouped in families sharing similar characteristics.

2.4 Axial development

A cartesian coordinate system that allows for localized coordinate definitions through linear mappings is widely implemented in computer graphics applications of CAD interfaces. The former is called Global, or World Coordinate System (GCS) and the latter is named Local or User Coordinate System (LCS).

In blade design and construction, the ideal Z axis of GCS is also referred to as "Pitch Axis". Airfoil sections lie on a curvilinear Blade Axis instead. Understanding the approximate nature of this convention is critical to be able to recreate the actual state of a decommissioned blade or establish a new design.

In short, the section curves defining the shell geometry lie in planes not necessarily parallel or aligned, and each host plane has its unique linear mapping of roto-translation from world coordinates. This nonlinear development is to accommodate for deflections and aerodynamic properties. The pre-bend serves to avoid collisions with the support tower under wind load, while the twist is conditioned for the large part by aerodynamic factors.

The coordinate transformations maintain $xi \perp yi \perp zi \; \forall \; \Pi i$ (orthonormal) and origin Oi $\forall \; \Pi i \in \gamma$ where the curve $\boldsymbol{\gamma}$ represents the blade axis. Within this definition the origin (Oi) of the plane can be an arbitrary point if it maintains a fixed relation to section geometry in each plane. We pick this point to be the intersection of the chord line and the line of maximum thickness perpendicular to the chord.

The assumption is that the resulting shell form of the turbine blade is constructed around a succession of airfoil profiles lying on planes arranged along the blade axis with plane normals tangent to the blade axis. These sections, remain in sequence if projected to the pitch (world Z) axis, parallel to the root (world X-Y) plane.

This, as previously noted, is one of the main differences from previously described segmentation methods. By specifying a curve instead of a series of offsets from and arbitrary pitch axis position we can reduce oblique plane problems into planar problems.

3. Blade Machine Point Cloud Processing:

3.1 Rebuilding Sections

In contrast to other methods such as rebuilding from technical drawings or tabulated data, point cloud segmentation starts with the spatial configuration already schematized. The cluster of points and related information is already geometrically tractable within a processing or modelling software. Human operators can approach the surveyed object intuitively for manual cleanup processes and automated geometric processing benefits from standardized formats of point cloud representation.

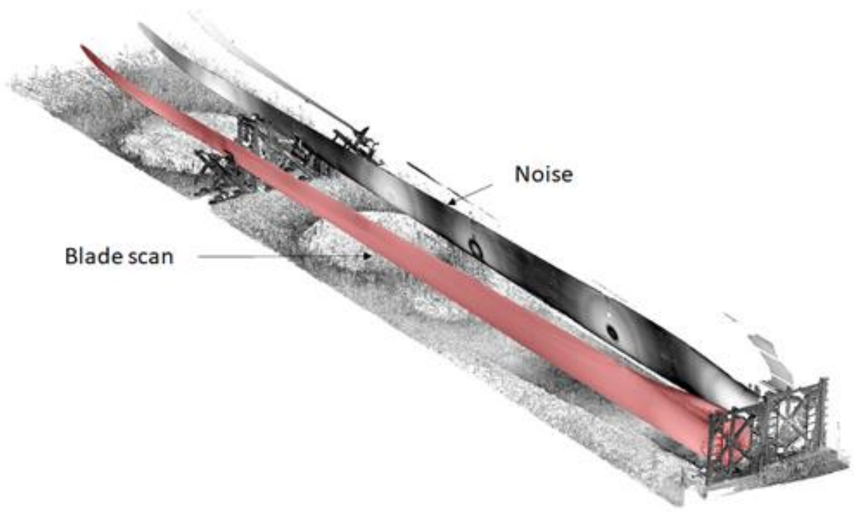

Figure 2: A Blade scan as stitched. Prior to cleanup.

As the geometries are distinguishable from other artificial or natural structures and color information can be hosted with the point cloud, the operator can easily identify the parts constituting the turbine blade proper and perform a manual cleanup. Once the turbine blade is isolated the cloud is imported in Rhinoceros 3D. Figure 2 displays the configuration of a field scan prior to cleanup.

The scan, depending on the configuration of the objects in the field, or initial registration of multiple scans can result oriented arbitrarily in the coordinate space of the CAD software. We rely on features common to turbine blades and reorient the point set to a conventional configuration for ease of visualization and simplification of later operations.

A typical sequence of operations can be summed up as follows:

1. Orienting the Point Cloud
2. Dimensional Reduction
3. Cross section reconstruction
4. Procrustean Matching

Throughout the process we rely on a series of geometric definitions repeatedly. These are convex hulls, minimum bounding boxes, and rectangles (MBB and MBR), and rotating calipers method to go from the former to the latter.

Initially we use rough approximations to find a bounding box around our point cloud that is representative of the dimensionality of our turbine blade. This is a rectangular prism with one dimension significantly larger than the other two. One definite way of finding the MBB is through the 3d extension of the convex hull and rotating calipers method (O'Rourke, 1985), yet a high density point cloud can cause this process to be onerous. As the time complexity for this method is cubic.

The iterative search for a small volume bounding box can be structured as an optimization problem that can start from and already reduced search space if the extreme points of the point cloud are taken and the initial bounding box is oriented through the two farthest pair of points. We can rely on an approximate search to find extreme regions to probe. We can then proceed to identify features of a typical turbine blade. The root and the tip. These two ends vary in the size of their imprint drastically and once we identify the root plane, knowing in which half space the tip is situated can provide us with a normal vector to align with the Pitch Axis, represented as the Z in the GCS of the CAD model space.

Following this, GCS oriented MBB edges provide us with directions by simple measurement of their lengths. The longest represents the pitch axis, the second one provides the direction of the maximum chord (to align with GCS X axis) and the shortest yields an approximate thickness. Similar searches in the extreme regions located along these yield, in the same manner as the root and tip, the LE-TE orientation, where the region corresponding to the LE will have a larger imprint than the one corresponding to the TE.

The interpolation of a good approximate for the blade axis lets us constrain our subsequent feature and geometry searches to planes. Planar searches reduce the complexity of operations and yield more clarity in the nature of the operations for the user.

Point clouds are sectioned by defining a plane and filtering out the points with a distance above a set threshold in the direction of the plane normal. This method does not account for duplicates and crowding, so a simple averaging of point coordinates yields results that are biased away from section centroids. Compounded with the variability of point densities between different regions in scans this can throw off any approximation. Thus, we execute a cleanup of the points.

Averaging points within a tolerance is useful in avoiding crowding and reducing the complexity arising from compaction of points within the intercepted volume. Once we reach sufficient homogeneity in distribution of points around the airfoil profile, we proceed to rebuild the section. Figure 3 indicates how centroids in different stages of reconstruction can show a bias due to scan artifacts.

3.2 Profile rebuilding

The point set is used to define the convex hull of the captured section. Convex hull is the smallest convex point set that contains all the members of the point set. The perimeter of this set provides us with valuable information used in matching our found airfoils to catalogued ones. First, the length of the perimeter is probed for distribution and average distance of points along the constituting edges. If there are aberrations in distribution the section is subject to corrective operations.

In a complete scan the longest edge commonly corresponds to the concave section of the HP side TE. This edge is then used to rebuild the concavity by way of a basis change. This secant is then replaced by the rebuilt concave profile to yield the airfoil shape. The points are then checked for deviations in distance from the concave profile and the profile is fitted within tolerance limits according to the average. Figure 3, shows a comparison of the convex and concave hulls and relative centroids.

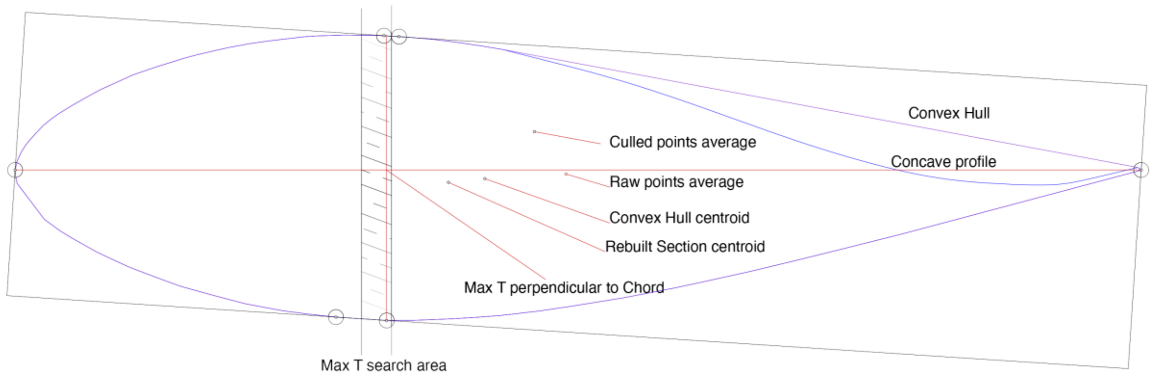

Figure 3: Rebuilt profile with Chord and Arbitrary MBR.

When the distribution of points indicates a lack of sampling for a given segment, the neighbors of the segment are used to interpolate a curve that will approximate the curve expected to be found in the region of the turbine blade. Once these gaps are patched, the search for the airfoil properties starts.

Our search relies on the observation that once the MBR is constructed around an airfoil there will be 5 or 6 intersections between this and the concave hull. The intersections along the short edges of the MBR yield the chord. As previously mentioned, while the MBR problem can be approached as an optimization problem with a clear minimum; with the two calipers method we can reduce our search space.

Freeman and Shapira (1975) prove that MBR must have one side parallel to one of the hull edges. This means that if we have a set of arbitrarily oriented MBRs in the direction of all the sides of our convex hull, the smallest area rectangle will be the absolute MBR. Toussaint (1983) provides a two-caliper approach that solves the problem in O(n) using two rotating calipers as this foregoes the linear search for MBR area at the end of the alignment procedure. The extension to 3D of this concept is not a naive search as the 2d approach. The MBB must have at least two adjacent faces flush with edges of the enclosed polyhedron. (O'Rourke, 1985)

Once the chord is intercepted, we redraw the MBR oriented along the chord to intercept a perpendicular chord and thickness combination. The second chord is not necessarily coincident with the first one yet after the second iteration we converge on the best approximation for the chord.

At this point we have a normal for the plane within which our section resides, and we have a chord and a reconstructed concave hull. Our set of airfoils to match with are organized and stored in a database where every entry has information with regards to its label, family of pertinence, Chord-to-Thickness ratio, position of maximum thickness projected on the chord, TE type alongside a polyline and a cubic spline interpolation.

All the geometric features are represented such that the chord of the stored airfoil is a unitized line starting at the LE and reaching TE. So, we can assume that every chord line we intercept from our sections is an applied vector on the LE-TE line, starting at LE. Thus, the length of the chord line is the uniform scale factor for the matching of known airfoil shapes to found airfoil shapes. Figure 4 displays information extracted from a very low-resolution scan.

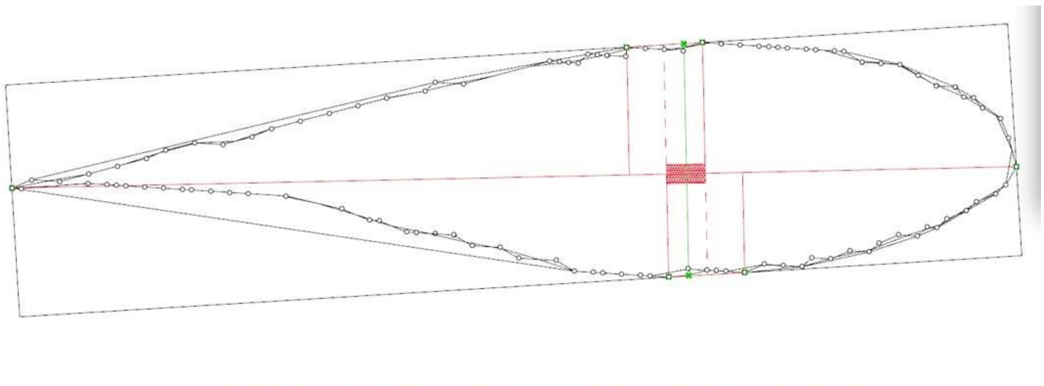

Figure 4: Depicting rotation and scaling factors derived from found section.

This kind of matching, where only translation, rotation and uniform scaling are allowed is referred to as Procrustean Matching. It maintains the shape of profiles that will be compared. In plane the operation requires a 2D translation vector and two scalars. One for rotation and one for scale scaling. The 3D extension of the same where we move our unitized profile to a plane along the blade axis requires 7 factors. One 3D Translation vector, another 3D rotation vector and again the scaling factor. This transformation, commonly used in geodesy is named Helmert Transform or seven-parameter transformation. Initial and final shape have a similarity relation.(Helmert, 1875, Helmert, 1876)

BM software has been developed with the possibility of going through each step manually. When the results of this automated workflow are compared with a manually matched turbine blade scan, the matches are compatible with what other researchers in the Re-Wind team have previously established. Figure 3 shows a comparison between values obtained by human or in different stages of automated search.

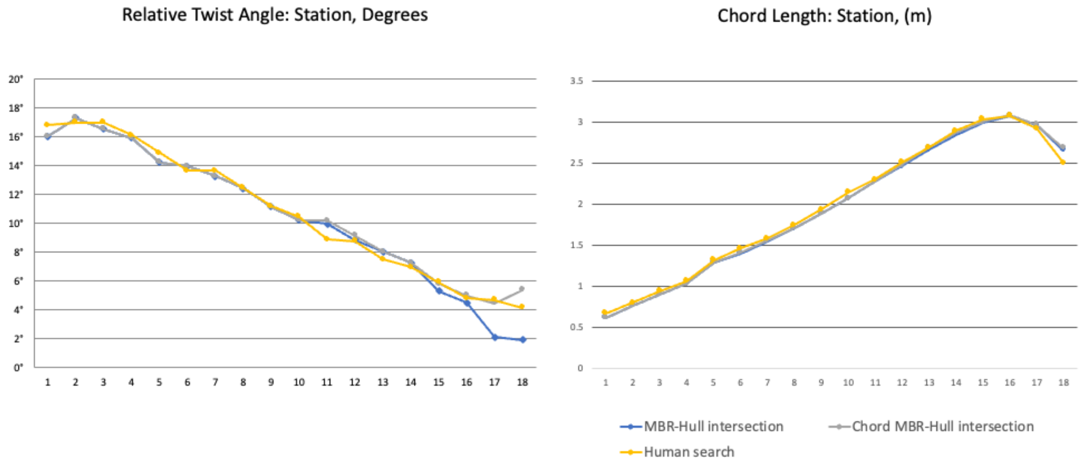

Figure 5. Comparison of automated and human searches for chords

3.3 Region Calculus

Having already extracted the Chord-to-Thickness ratio, through comparison of our chord oriented MBR's sides, we can filter our database for airfoils that have similar proportions. This is a numerical operation to help limit the number of candidates to which will have to apply transformations and region algebra to find the closest match.

Establishing objective comparison parameters are essential to evaluate similarity between shapes. Expressed in Region Connection Calculus (RCC) terms the relationship between shapes being matched present Partial Overlap (PO) or more rarely Tangential Proper Part (TPP and TPPi) relations. (Randell et al., 1992)

For each comparison, following the uniform scaling and alignment along LE-TE coordinates, we extract a series of resulting regions. Assuming Region A is the found airfoil and Region B is the candidate, we calculate the area of the overlap ($A \cap B$) and overflow ($A^c \cup B^c$). In DE-9IM terms these correspond to $I(a) \cap I(b)$ for the former and $I[(a) \cap E(b)] \cup [E(a) \cap I(b)]$ for the latter. Figure 6 shows PO conditions with 3 different candidates.

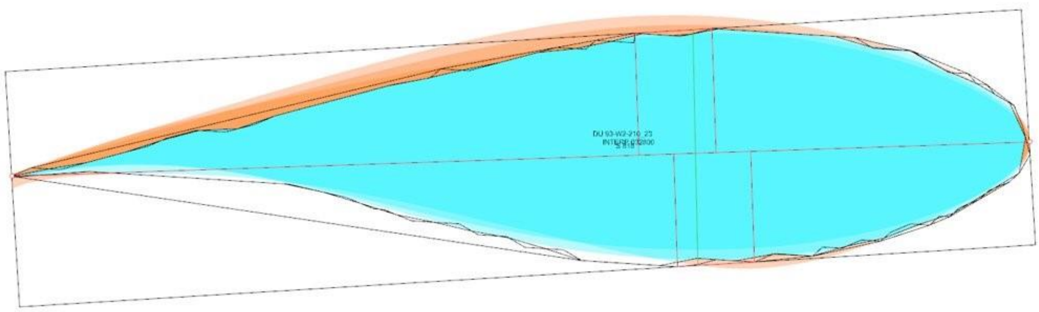

Figure 6: Demonstrating intersection or overlap in cyan and union of complements or overflow in orange

The selected candidates are sorted by a ranking where overlap area proportional to the area of shape A is a bonus and the overflow areas proportional to the area of shape A are penalties. The best fit airfoil is then selected and roto-translated to the initial position of the section in the GCS of the CAD system to rebuild the turbine blade hull.

The final configuration is output in a geometric model, an AeroDYN formatted spreadsheet and a list of blade axis tangent vectors at a pre-set location within the section.

4. Conclusions and Future Work:

Rebuilding of the idealized shape is useful for a global structural analysis, and large-scale volume representation. These can be utilized for property analyses to establish feasibility of re-use for given applications.

Through a comparative analysis of the idealized shape and the actual point cloud scan, we can identify damage, deformations, and missing sections. Idealized shape model can be also used as a substrate to encode additional information such as unexpected artifacts. Once the ideal form is constructed production of a planar mapping to annotate visual observations is a trivial task.

The modular dataflow provides the possibility of alternating or substituting rule-based automation with human operators. With the limited amount of information given the current level of necessity in reverse engineering turbine blades it is not very feasible to imagine a vast availability of examples. Making use of these recorded sections can be valuable. In the future a systematic collection of these errors and results can constitute a training set.

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